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Decision-Making Using Mathematics

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Abstract

This paper presents a mathematical decision-making tool that is easy to use and express mathematically, requires only a little mathematical expertise, and gives results that are easy to interpret. The tool has been used extensively as part of a term project in a precalculus class. The decision-making process forces students to consider the Christian ideals of faith, morals, and ethics, and to use them in a quantitative form in order to make their decision. The overall success of the tool comes from the integration of several different kinds of student learning experiences in a properly designed project. That is to say, the student must look deeply into his or her own core values in an effort to quantify his or her feelings toward God, faith, morals, and ethics. This decision-making mathematical tool is something that students can take with them as they journey through life. It is the primary objective of this paper to identify the tool, to show how it is used, and to encourage other Christian educators to use it in their classes too.

Introduction

The decision to introduce a faith-based learning project into a precalculus class stemmed from attendance at a faith and learning seminar offered by Whitworth University. Importantly, the project was to be centered around a mathematical decision-making tool that could easily be used by non-math major students. Further, the decision criteria were to include faith in God, morals, and ethics. To accomplish this, two things needed further investigation. First, can mathematics be truly integrated into a faith and learning project? And second, what kind of mathematical tool should be used to make a decision?

A paper edited by VanBrummelen (1999) caught my attention. It states that Christian ethical principles such as righteousness, justice, and respect for the dignity of human form are an indispensable basis of a functional society. The article purports that faith-based learning intends to lead students to know God and His world, and that the impact of that vision depends upon our faith, integrity, and passion. Students need to develop a clearer understanding of their own lives and how they can put their deepened faith and knowledge into action. During this process, it is obvious that they will have to make many faith-based decisions.

Rigelman (2003) responds to the question "How will I know if the [faith and learning] integration strategies I am employing are meaningful and effective?" with the following answer: They will be, if values, choices, decision-making, and ethics are central to the work of a Christian teacher.

Barker (2004) argues that mathematics, like any other discipline, can be integrated into a faith and learning atmosphere. He states that it is not a matter of connecting math to theology as academic disciplines, but rather of connecting the particular Christian vocation of mathematics to the universal Christian vocation of love and service. Here again, making decisions based upon faith, morals, and ethics is a positive step in the correct direction.

An often referenced book that deals with mathematics from a Christian perspective is one by Howell and Bradley (2001). It is a comprehensive volume that begins with a history of

mathematics that leads to how math has influenced modern science as well as culture. This material sets the stage for a discussion of the faith perspective of mathematics. Howell and Bradley state, "We are creatures of the Lord meant to experience our analytical and quantitative abilities in the service of other people and the rest of creation." They point out that our analytical abilities were not given us to further our own ends. If we can use them to help others lead a happy, meaningful life, then we are using our abilities as they were meant to be used.

Mathematics, over the centuries, has played a role in music by helping us understand the blending of various harmonics into pleasantly sounding chords. It has played a role in art by giving artists an understanding of aesthetically pleasing ratios. Mathematics is the necessary tool for engineers to build buildings, bridges, and communications systems. All of these things have made life more pleasant for many others.

For a Christian, mathematics does not exist in isolation from other aspects of reality. Mathematics, when used properly, should fill us with a sense of gratitude and appreciation for the analytical talents God has made available to us. It should make us glad that we have the capabilities to increase the quality of life and make life more enjoyable for all. The mathematical decision-making tool presented in this paper, if used properly, should lead to faith-based decisions, which in turn should lead to a higher quality Christian life. With this in mind, mathematics will be playing its proper role as one of our talents.

Sometimes a mathematical model requires human senses to be our inputs, and they are not considered as infallible devices. These devices are then placed in a strictly defined manner to record specific information. Herrmann (2007) reminds the reader that in order to construct a mathematical model, physical statements must be expressed in the same manner as the mathematical theory so that they can be interpreted. In the case of the faith, morals, and ethics of this paper, we use numerical values to describe our depth of feeling associated with them.

Methods and Materials

It was previously stated that the original concept was to develop a faith-based learning project for a precalculus class. A class like this may be the only math class that a non-math/science/business student takes. As a consequence, the mathematical tool cannot be beyond the student's mathematical sophistication level. With this in mind, several mathematical tools were considered. The selection requirements were as follows: (a) it must be easy to use and easy to express mathematically, (b) it should not intimidate non-mathematically oriented people, and (c) the mathematical result of the decision should be easy to interpret. By satisfying these requirements, the decision-making tool should gain in overall acceptance and use.

Integer Programming is the study of optimization problems where some of the variables are required to take integer values (Fletcher, 2001). The branch and bound method is one of these and is of wide generality, yet reasonably efficient. It does, however, require a user who is adept at mathematics beyond the precalculus level. It does not satisfy the second requirement listed above, and so it was rejected. Geometrical programming and network programming were also considered but also rejected for being too complex to satisfy the second requirement.

A decision matrix was next considered as a possible choice of method. A decision matrix is also known by many other names, Pugh matrix, decision grid, problem selection matrix, and criteria based matrix (Tague, 2005). It is often used when a list of options must be narrowed to one choice using several criteria and where the number of options is of a manageable quantity. While the decision matrix sounds as if it might be a good method to use, it too would require

more mathematical expertise than generally found in a precalculus class, and the second requirement would not be satisfied.

Finally, a series approximation method was considered (Pierre, 1969). It appeared that with the proper adaptation, in the form of a weighted average, this method might well satisfy all three requirements. However, one other method was examined before pursuing this approach.

Business managers often use a decision tree approach to make a decision as to which business strategy to choose. Verma and Gross (1978), among others, discuss this method. The shortcoming of this approach lies with the strategist having to make an educated guess of future asset values and probabilities of events. The future is not known, so one must accept educated guesses as part of the solution process. However, if not enough attention (i.e., due diligence) is paid to establishing future asset values and probabilities, then the decision will be flawed. The method presented in this paper is quite different from decision trees. Nevertheless, it does share some common ground with that approach. It too, requires some educated guesses in order to accomplish its task. Here again, a great deal of introspection must be done in order to assure unflawed results.

Now to return to how an adaptation of a weighted average could be used as a decision-making tool:

Triola (2007) writes a weighted mean as $\bar{x} = \frac{\sum(w \cdot x)}{\sum w}$. Here $\bar{x}$ is the weighted mean, w is the weighting factor, and x is the parameter under consideration. For example, assume its use is computing a student's average test score where the tests have a difference of importance. First the product of each individual test score (x_i) and its weighting factor (w_i) are determined and then all of these products are summed and divided by the sum of the weighting factors. Clearly this is not a threatening tool for a non-math student to use and it does satisfy all three requirements quite well.

The mathematical decision-making tool presented here starts with the weighted mean expression, but has several important variations. As such, it will take on a somewhat different form. First, by requiring $\sum w=1.0$, we can eliminate the need for this term. Next, the w_i (weighting factor) will be replaced by a numerically prioritized 'decision criteria' and lastly, the x_i will be replaced with a numerically determined impact value. All of this is explained in the next section.

Methods

The introduction of this paper pointed out that the purpose was to put together a faith-based learning project to be used in a precalculus class. The project would be centered around a mathematical decision-making tool. The project should be such that the student would have to make a decision, and the decision criteria would have to include faith in God, morals, and ethics. The student would make his or her decision using the decision-making tool as described later.

In order to focus the term project, each student could select one of several possible projects that places him or her in an environment sometime in the future. The students then have to use the decision-making tool to make their decision. To accomplish this, students must look inside themselves in an effort to determine how they truly feel regarding faith, morals, and ethics, as well as any other decision criteria. Next, each student must apply the knowledge gained

from the introspection of their core values because they have to quantify their feelings. An example project environment is described below:

- They have graduated from college.
- They are married with two children.
- They are working for a company and like both their job and the company.
- Their company has recently acquired a smaller company.
- They have been selected to be part of a team to increase synergy after the acquisition.
- They are given free access to all records.
- They have discovered some highly secretive information.
- The information is considered as illegal, immoral, and unethical by most standards.

Having uncovered this secretive information, and not knowing who, if anyone, in upper management is already aware of this information, they must decide on what action to take. They are given four choices of action/responses:

a. Do nothing and ignore or shred what was found.
b. Ask for their old job back.
c. Report their findings to management.
d. Be a "whistle blower" to the outside world.

In order to make a choice, the student must determine how the action taken would affect things—the decision criteria—that are important to them. They are as follows:

1. Self-preservation (employability and credibility)
2. Company loyalty
3. Faith
4. Morals
5. Ethics

To use the mathematical tool, one must now determine the relative importance of each of the decision criteria; therefore, each student's first job is to prioritize and then to determine the weighting factor of each criterion. This is a very important step because the student must determine how they truly feel about the importance of these things. If this is not done with great care and assiduity, the overall results of the decision-making tool will be flawed. Said another way, if the inputs to the models are not good, neither will be the outputs. In order to make it simple, it is required that the sum of these weighting factors be 1.00. This constraint eliminates the need for dividing by $\sum w$ in the weighted mean ($\bar{x}$) expression. Furthermore and very importantly, the weighting factors do not change with the choice of action (response) to be taken.

Next, a parameter, called an impact value (v), is introduced, which simply indicates how the selected choice of action will impact each of the decision criteria. For example, if the action taken (response) is (a) do nothing and ignore/shred what was found, how does that affect the student's relationship with God (i.e., faith) or company loyalty? As such, each student must compute a set of impact values for each course of action (response). This step requires a great deal of introspection too, as students must again look inside themselves and determine how they truly feel. As with the determination of weighting factors, if this is not done in an assiduous manner, the results will be flawed. For purposes of the project, the impact values vary between 0

and 10. If self-preservation (the first of the decision criteria) were assigned an impact value of v=10, that would mean that self-preservation was not affected by the course of action under consideration. However, if self-preservation were assigned an impact value of 5.0, it would mean that self-preservation would be significantly impacted by this course of action (response). In other words, the decision maker might be demoted and/or lose significant credibility within the work force. An impact value of 2.0 would mean that self-preservation would be even more significantly affected by this response. These changes transform the weighted mean expression to the following:

$$\text{RS (Response Score)} = \sum(w \cdot v)$$

where w = the weighting factor(s) and v = the impact value(s). The summation $\sum(w \cdot v)$ thus would contain five terms since, for this example, there are five decision criteria (listed above). A response score (RS) must be computed for each of the four possible actions/responses that could be taken. That means that the student must compute four response scores (RS), one for each of the possible actions/responses listed above. The course of action with the largest response score is the proper choice of action (response) because the student would want to choose a course of action that minimizes the collective impact on the decision criteria.

If one thinks of the five decision criteria as all being equal in importance, then each weighting factor would be 0.20. So if a particular criterion is considered somewhat more important, then its weighting factor should be greater than 0.20. How much greater depends upon how strongly one feels about it. The stronger the feeling, the more above 0.20 the weighting factor should be. If, however, a criterion is considered somewhat less important than the others are, then it should have a weighting factor less than 0.20. It is important to recall that the sum of the weighting factors is 1.0. This means that if one weighting factor is made greater than 0.2, another must be made less than 0.2. It should be remembered that the weighting factors do not change. That means they are the same for each possible course of action (response).

Suppose a student very thoughtfully assigns weighting factors for the decision criteria and chooses the weighting factors to be the following:

Self-preservation	0.15
Company loyalty	0.20
Faith	0.30
Morals	0.25
Ethics	0.10
	1.00

Next, make the assumption that in considering response (b) “ask for your old job back,” a student arrived at a set of impact values as follows:

Self-preservation	6
Company loyalty	7
Faith	8
Morals	8
Ethics	8

Using these two sets of values, one can compute a response score (RS) for the action (b) “ask for your old job back” as follows:

$$RS = (0.15)(6) + (0.20)(7) + (0.30)(8) + (0.25)(8) + (0.10)(8) = 7.50$$

As can be seen, the response score for "ask for your old job back" is 7.50. It is important to realize that while the weighting factors never change, the impact values for actions a-d will not take on the same value(s). That is to say, impact values vary depending upon the course of action (response) being considered.

Next the student must repeat this procedure and obtain a response score for the other three courses of action/responses: "do nothing and ignore what was found," "report your findings to management," and "be a 'whistle blower' to the outside world."

Lastly, the student must compare all response scores and select the largest as his or her course of action (response). Again, the reasoning is that the largest RS means less impact on the overall decision criteria. To see this more clearly, one needs to consider what these response scores mean. If the perfect decision was made, one would have chosen the perfect course of action (i.e., a course of action that did not impact any of the decision criteria). As such, one would have rated each of the impact values for each of the decision criteria as a 10. The response score would have the following appearance:

$$RS \text{ (perfect decision)} = (0.15)(10) + (0.20)(10) + (0.30)(10) + (0.25)(10) + (0.10)(10)$$
$$RS \text{ (perfect decision)} = 10.0$$

Courses of action are rarely perfect and, as such, each of the decision criteria is usually impacted. That is, in this example, one must compromise on self-preservation, faith, etc. The more compromise from a perfect course of action that the student must make, the lower the impact value (of the various decision criteria) will be. While a student may not have the perfect course of action, he or she should choose the one with the least amount of compromise on the decision criteria. Therefore, a student should choose the course of action (response) with the highest response score.

It should be noted that with the limitations suggested on the parameters, the RS will fall between 0 and 10. Also, it should again be recalled that the sum of the five weighting factors is 1.0 and that once the weighting factors are selected, they are the same for all choices of action.

Students usually adjust both the weighting factors and the impact values to get an idea of the effect they will have.

Selected Student Responses to the Class Project

This class project has been included as part of my precalculus classes for two basic reasons (a) to introduce the mathematical decision-making tool and (b) to give each student an opportunity to better know themselves and their core values.

The formula presented here is simply a generalization of a weighted mean expression. This term project is appropriate for an intermediate algebra class, a precalculus class, or an elementary statistics class to name a few.

The decision criteria (through determining weighting factors and impact values) force the students to look into their own beings and to go so far as to quantify how they truly feel about such concepts as faith, morals, and ethics. They have to better understand their relationship with God and then have to determine their own balance between these concepts and their own self-preservation and (in this example) company loyalty. It thrusts their thoughts and reasoning into an adult world of which they are only approaching. They must advance themselves by

introspection into their true feelings. I have used this term project numerous times in my precalculus classes and have had the opportunity to collect a few student comments, which follow:

> College students are widely believed to be the self-centered children of the academic world. In many ways, this is true. We spend much of our time focusing on our own needs, and our own education. However, I still have a difficult time examining my core values and belief system. In the course of completing this project, I've found that I don't know myself very well at all. For example, I had to reference the definitions of "faith," "moral," and "ethics" several times. Personally I had different definitions for each word, which I now can see the flaws in. When tackling the question "What would you do?" for the first time, I was stumped. I honestly had no idea how to put myself in that position. How was I, a humble college student, supposed to identify with an adult, with responsibilities in the workplace? However, I soon realized that it was better that I work through this now, than when I had to in the real world.

Another student had this to say:

> Though my adult life has only recently begun, I have already been required to make some difficult, life-changing decisions. Although an algebraic formula is not typically my first resource for assistance in making an important decision, I can understand how a mathematical decision-making tool could be useful in providing substantiation for a difficult choice. In utilizing this specific formula, I was able to assess my own values and come to the conclusion that, like many young adults, I may be too motivated by self-glory and financial security, and not yet secure in my personal faith and other beliefs. This discovery has encouraged me to become stronger in these vital parts of my being and to reevaluate my priorities in life.

The next comment is more directed at the tool itself.

> I believe this decision-making tool would be useful in decisions that have outcomes numbering more than just two. For example I can see where I would use this tool to decide between summer jobs or internships when weighing possible future benefits, or even something as abstract as where to spend your money based on what you know about the company practices. By inserting different or even more categories besides self preservation, faith, morals, etc., I believe the tool could have an almost unlimited application to decision-making.

This term project can also awaken or further Christian ideals as evidenced by this comment:

> Doing this paper for math was a thought-provoking exercise. It made me think about my beliefs and what is most important to me. This method of making decisions is very helpful and there are many ways that I can use it to help me both now and later in life: picking out a major and deciding on a job after college are just a couple of them. Although I will need to do other things to help me make such significant, life-changing decisions, this mathematical formula is one method I can use to help me discover God's will for me in such important choices. I think that God gave us logical, reasoning minds so that we can use them to help us discover what he wants us to do. "Be very careful, then, how you live—not as unwise but as wise, making the most of every opportunity,

> because the days are evil. Therefore do not be foolish, but understand what the Lord's will is." (Ephesians 5:15-17)

The next comment describes wrestling with quantifying one's feelings about such abstract ideas as faith, morals, etc.:

> In this decision-making project, I gave self-preservation a weight of 0.1, company loyalty a weight of 0.1, faith a weight of 0.3, morals a weight of 0.2, and ethics a weight of 0.3. This was probably the hardest step for me. It was difficult to put a numerical value on things I had never consciously thought about before. I have thought about making a tough decision and looked at different aspects of my life that would be affected, or that would affect my decision, but never these specific aspects.

The same student then continues the thought process:

> Assigning a weight to faith, ethics and morals was more difficult because I was not quite sure how I separated them in my mind. For me, ethics and morals are usually grouped into the same categories. Also, faith is a major determinant for what I think is ethical and moral.

Another student had this to say:

> This project was very insightful to us, because it put us in a position where we had to evaluate what decisions we would make if it came down to choosing between faith and self preservation. We should always put our faith first because God is our priority, and we should strive to live for Him.

This student then followed that comment with this:

> We learned a lot by using the math tool. I liked it because it combined logic with everyday decision-making. It is something that I could see myself using in the future to help organize and sort thoughts in order to determine the best course of action. At first I was a little baffled by the importance of this decision-making tool, but after reviewing the criteria and practicing it, I came to the conclusion that it is an essential tool that can be utilized in making decisions regarding school and career choices.

The final thoughts of one term paper about the decision-making tool were expressed in this way by one of my students:

> After doing this project, I have come to appreciate this tool as helpful to use when confronted with a difficult decision. As for me, it helped me get a better grasp on my own values and principles. These are essential to consider when contemplating the various ramifications associated with a decision. However, I would not wholly depend my final decision on this tool. I think this is a good way to get an idea of what choice would be best for me, but I do not think I should exclude prayer from the equation.

The next comment is from another student primarily focusing on the tool itself:

> This tool has helped me to understand the importance of weighing all consequences of decisions we make in our lives, though I feel I could have easily discerned the course of action I should take by consulting my gut feeling. This tool has helped me to understand how looking at every aspect of a problem can foster the most wise decision in times of confusion, which will foster no regret as it shows which course will leave one's integrity and overall well-being most intact.

As a final comment, this next student sees a blending of the mathematics and the intuitive:

> It is very interesting how someone's beliefs can be shown in the numbers and equations, but they can, as proven in this project. The equation lines up almost perfectly with how I feel about the situation. The weighted factors are a part of everyone's life and determine the outcome of almost everything that someone does.

Conclusion

The mathematical decision-making tool presented in this paper, if used properly, has the ability to lead to a variety of faith-based decisions. These in turn should lead to a higher quality Christian life. If a project such as the one described here is added to a math course, it integrates several different types of student experience into the learning environment. The students not only learn how to use the tool, but also have the opportunity to investigate their core beliefs thoroughly. This happens when they prioritize and quantify the decision criteria. It happens again when the impact values are established. The project environment itself (which may be varied) can add to the student experience. The learning diversity of a project like the one described should prove to be of great benefit to students in the classroom.

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