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Teaching calculus with Wolfram|Alpha

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This article describes the benefits and drawbacks of using Wolfram|Alpha as the platform for teaching calculus concepts in the lab setting. It is a result of our experiences designing and creating an entirely new set of labs using Wolfram|Alpha. We present the reasoning behind our transition from using a standard computer algebra system (CAS) to Wolfram|Alpha in our differential and integral calculus labs, together with the positive results from our experience. We also discuss the current limitations of Wolfram|Alpha, including a discussion on why we still use a CAS for our multivariate calculus labs.

Keywords: mathematics; calculus; Wolfram|Alpha; CAS; labs

1. Introduction

Wolfram|Alpha is a free, browser-based web service, developed by Wolfram Research, which dynamically calculates results to natural language queries by applying algorithms to its extensive internal database of facts. Users submit queries or computation requests such as 'What is the half-life of Strontium-90?' or 'What's the derivative of x^x?' via a standard one-line text box. Wolfram|Alpha responds with data and computed results in textual, symbolic and graphical representations (Figure 1). While Wolfram|Alpha can be used for much more than calculus, this article will detail our view and experience of using it to teach calculus, and hereafter our views should be taken within that context. A more detailed discussion of its general uses can be found by reading the official Wolfram|Alpha blog and forum available through the official Wolfram|Alpha website [1].

Since the launch of Wolfram|Alpha in May 2009, much discussion has taken place regarding its possible impact, both positive and negative [2–6]. While some professors were at first leery of its 'show details' ability, the majority of members of the mathematics community who have experimented with Wolfram|Alpha are encouraging students to utilize Wolfram|Alpha to assist with homework or other general computational questions. One example of this can be seen by viewing the short video 'Wolfram|Alpha for Calculus Students' created by Talbert [7]. For instructors not familiar with Wolfram|Alpha, The Walpha Wiki [8], created by Derek Bruff, is a way for individuals to see specific examples of how Wolfram|Alpha might be used in a variety of math courses, including upper division courses.

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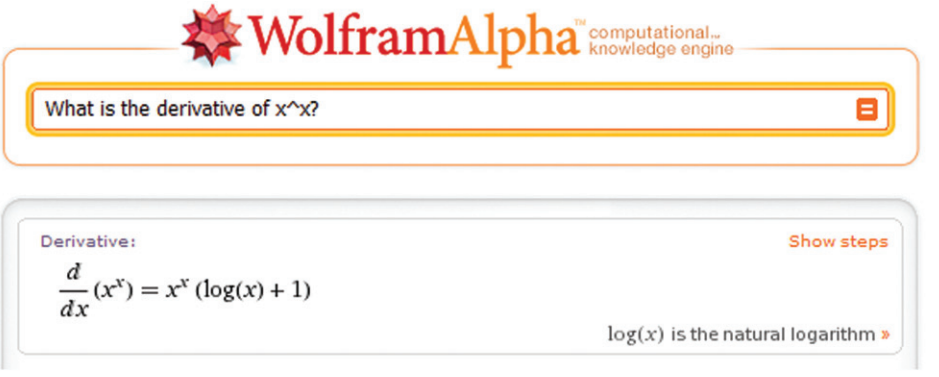

Figure 1. What is the derivative of x^x?

This article addresses our experience of using Wolfram|Alpha to replace an existing computer algebra system (CAS). In each of the courses in the Calculus sequence, students are required to attend a lab session 1 day per week on which they work through an assignment using a CAS. The lab assignments are meant to provide an extension of the material discussed in the class, and the use of a CAS allows for both visual and computational connections that are more difficult to achieve by hand. We have included our new Labs as a supplementary file, available online under the 'Supplementary Content' tab on the article web page or from the author on request. This type of lab experience has become common among Calculus courses at colleges and universities [9].

After assessing the current college scenerio, several key issues were noted. First, decreasing budgets make it more difficult to keep up with the annual upgrade of CAS software. Second, the college campus is experiencing an increase in students who live off campus as well as students who are choosing to take courses online. Hence, travelling to campus to a computer lab equipped with the latest version of the CAS is not only an inconvenience but may not be an option too. Finally, during the past several years, we have noticed that students mainly get bogged down with syntax errors and never actually experience the original purpose of the lab assignment. With these issues in mind, the question that arose is, can we replace the current CAS with Wolfram|Alpha and still achieve all our goals for the lab experience? Even more, could it be possible that Wolfram|Alpha would increase the effectiveness of the lab assignments?

2. Why Wolfram|Alpha?

During the initial unveiling of Wolfram|Alpha, there was much discussion about the effect it might have in college math courses. The discussion sparked the thought, how can students use this free resource to the benefit of both student and instructor? Knowing that behind the scenes Wolfram|Alpha is powered by Mathematica, the lab component of the calculus courses offered an obvious and exciting possibility. In an effort to make the project most successful, the calculus team contacted Wolfram

Research to enquire about the best way to approach the project and in turn a collaboration was born. The Wolfram Research team has been very willing to receive suggestions and address any issues that we had and always had an answer for us within 24 h.

Some may be concerned about the limitations involved in not using a full CAS. In the beginning calculus courses, students are not required to use many of the more powerful components of a full CAS such as Mathematica. In fact, it is reasonable to say that they are not in a situation where they can take full advantage of such a system. This made it practical to consider that Wolfram|Alpha might be enough for these courses. The possibility of reducing campus IT involvement with installations and upgrades, and reducing the cost of CAS upgrades made the project worthwhile.

Another need that fed the excitement of using Wolfram|Alpha was the opportunity for students to have access from anywhere. Students were no longer limited to lab hours of operation or number of computers. A decrease in the amount of student lab classrooms available is a large concern in our department, something that is also a concern at many institutions [10]. Utilizing Wolfram|Alpha meant that students could enlist their own computer (or iPhone) at whatever time was convenient. This would also allow for the possibility of online students to participate in the lab assignments.

A significant advantage of using Wolfram|Alpha over other full CAS is the reduced emphasis on syntax. Students can enter queries using the same language they would use to ask their professor a question. Many hours have been wasted in the lab due to missing parentheses or semicolons. Wolfram|Alpha allows students to actually focus on the topic at hand and not on the syntax.

In addition, Wolfram|Alpha also provides more information than a one word (or number) answer to the student's original question. Often students may not be sure how to request certain output or possibly do not know how to even ask the question. So the fact that Wolfram|Alpha will report all knowledge from its database in a variety of formats is an advantage for the student. One simple query can result in an equation, a graph, an exact answer and an approximate answer. Even more helpful for the student is the ability of Wolfram|Alpha to display the steps it used to arrive at a solution. The student then has an opportunity to view the process as well as the final answer. Sometimes Wolfram|Alpha chooses a more rigorous process than one might expect due to its algorithmic nature of using Mathematica behind the scenes. However, most examples result in an explanation that is useful for the student (Figure 2). The 'Show Steps' feature of Wolfram|Alpha is one clear advantage that it has over Mathematica, especially since there are no plans to introduce this feature into Mathematica itself [11].

Moving away from a full CAS, implementing labs with graphing calculators may seem appropriate, as often times students are already familiar with the technology. While possible, we feel that using Wolfram|Alpha is far more practical, at least for labs. The output from Wolfram|Alpha, including graphs, is easily copied and pasted into a student's lab assignment which they turn in each week. For certain graphics and demonstrations, one can easily go back and forth between Wolfram|Alpha, MathWorld [12] and Wolfram Demonstration Project [13]. With Wolfram|Alpha, there is no need to make sure everyone has to buy the same calculator; indeed there is no cost for the student, though they may pay for the iPhone App [14], and with Wolfram|Alpha every student is using the latest version. Wolfram|Alpha is also, in

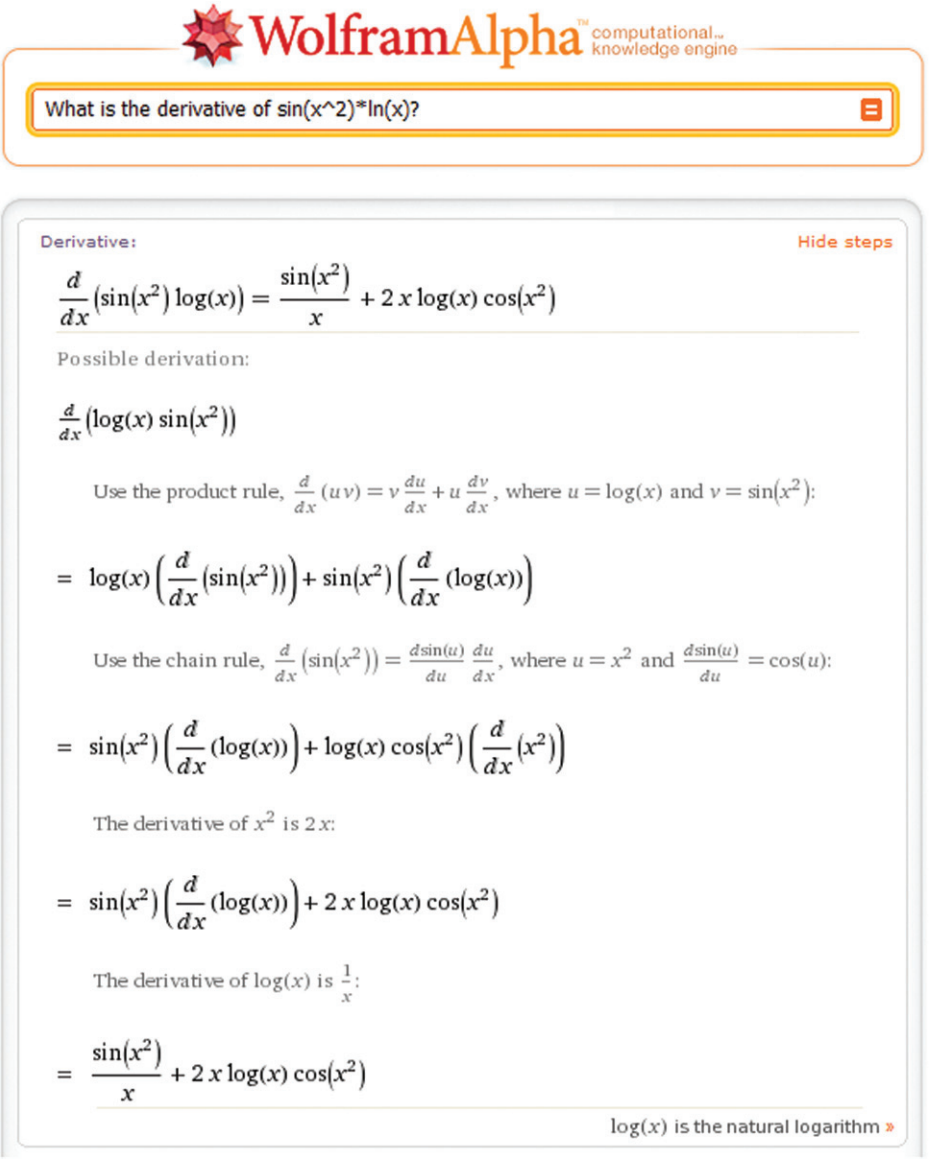

Figure 2. Using the 'Show Steps' feature.

general, more powerful than a graphing calculator, though calculators at present are superior when it comes to windowing and zooming into graphs. For example, it is currently difficult to specify aspect ratios and ranges for graphs in Wolfram|Alpha.

While most students are more familiar with graphing calculators from high school, we see this paradigm shifting too, as does Wolfram Research [15]:

> '*My bet is that most high school and college students will find the Wolfram|Alpha App most useful for helping understand homework assignments. In addition to enabling more complex computations, the supercomputer cloud also allows the app to provide more than just the answer—such as various plots of the result, additional or alternative forms of the*

solution, the steps needed to solve the problem, and much more. As shown at Homework Day, this additional information that "surrounds" the answer can be incredibly helpful in better understanding the problem and the solution' [14].

3. Calculus One – differential calculus

The lab assignments created for Calculus One (in this case that would include vectors, limits, and derivatives and their applications) were based upon our previous year's CAS labs and also sometimes adapted from exercises from the text that were marked as calculator or CAS problems. The students came from a variety of science-related majors and were on the majority freshmen. From a show of hands, it was apparent that none of the students had any experience with a CAS and little experience with Wolfram|Alpha. As a professor with 18+ years of teaching calculus with a variety of CASs at several institutions, this came as no surprise. As expected, the tool that most students had experience with (and nowadays seem to turn to even before thinking) was a standard graphing calculator. Indeed for the first few labs, the students would go back and forth between their notes, the text, Wolfram|Alpha and their calculator. As the semester progressed, students became more comfortable with Wolfram|Alpha and I began to see less and less use of calculators, though they continued to be their security blanket during lectures. Since Wolfram|Alpha at the time of teaching was only a few months old, this was very encouraging. Over the next 10 years, we foresee the level of familiarity with Wolfram|Alpha that incoming freshmen have will steadily increase.

The labs that we had previously developed for Calculus One using a CAS began with examples and explanations that the students were expected to work through and then finished with a series of questions meant to increase student understanding of certain concepts by leveraging the power of the CAS for graphing and animation, etc. It was our experience that as we went around helping the students, more often than not and especially for Calculus One, we were fixing student's syntax rather than explaining concepts.

A remarkable change occurred when we switched to using Wolfram|Alpha. Since students could now use a more natural language to input their queries, I spent less time explaining syntax and more time explaining concepts. In fact, I had to think carefully how to phrase the lab questions so that the students could not just copy and paste the questions directly into Wolfram|Alpha and get the answer immediately. Though this project was never meant to be an educational study, I can report anecdotally that the labs were now doing exactly what they were designed to do, and that was to reinforce concepts talked about in class and have students discover for themselves solutions to complex questions and applications.

As mentioned previously, that fact that Wolfram|Alpha is platform independent and is always up-to-date (and free) means that students are not tied to a lab or even a computer; several students were using netbooks, and one student actually did a lot of the work on his iPhone. I also intend to use Wolfram|Alpha as a demonstration tool during class.

As to the problems I had, most were resolved very quickly as we communicated with Wolfram Research, including a bug that one of my students found. For example, evaluating a derivative at a point was at first possible but could only be attained by using syntax that was not natural (for the student). This issue was

quickly fixed by Wolfram|Alpha. The only outstanding issue that I experienced is the issue of controlling aspect ratio and the exact window for certain graphs. This does not seem possible at the time of writing, even when using exact Mathematica syntax.

Overall, I feel that using Wolfram|Alpha in Calculus One has immediate advantages over a standard CAS, and as Wolfram|Alpha continues to improve, I only see the advantages increasing.

4. Calculus Two – integral calculus

The lab assignments created for Calculus Two (in this case that would include integral calculus with applications, sequences and series) were designed with the purpose of using Wolfram|Alpha as a computational tool to enhance conceptual knowledge. Hence, the exercises require the student to demonstrate ability beyond the levels of knowledge and comprehension and move towards higher levels of Bloom's Taxonomy of cognitive thought including analysis and comprehension. Lab questions are formulated so that students are asked to compare, test, synthesize and predict. Creating lab questions with this in mind provides an opportunity for a knowledge engine such as Wolfram|Alpha to be used as a tool for the student to be able to formulate inferences instead of simply reporting output. In most lab assignments, the student was also required to show the simple output so that the professor can 'see' what the student used as a reference.

One example would be a question about the area under a curve. The lab assignment would ask the student to evaluate several definite integrals. Since Wolfram|Alpha displays both the numeric result and the graphical interpretation, the student is able to see connections in how the integral is set up and how the value of the integral changes based on the limits of integration (Figures 3 and 4). Once the student has an opportunity to observe examples, then the student will be required to make predictions about the result of other definite integrals. Questions can address concepts such as signed areas, symmetry and area or absolute value of areas.

It should be noted that there are some concepts that Wolfram|Alpha cannot address directly such as approximation techniques for integration. Wolfram|Alpha will not return a numeric or graphical result from a request to approximate an integral using the trapezoid or Simpson's rule. However, for many of these common calculus concepts, the student can access the link to the Wolfram Demonstration Project [13] from the Wolfram|Alpha site or visit MathWorld [12], another Wolfram creation. In the case of approximation techniques, the student can go the MathWorld site, type 'Riemann Sum' in the search box and have access to a player that will allow them to input their function, the interval, number of subintervals and select the approximation method (Figure 5). In order to view the activities listed in the Wolfram Demonstration Project, the student is required to download (at no cost) a Mathematica Player. If the student is using their own computer, this is not a problem; however, if you are in university lab classroom, there may be some information technology (IT) issues with allowing students to download freely.

Each lab assignment required a written component in which the student would summarize or analyse results instead of simply collecting source code, which often occurs when a full CAS is used. Most students chose to use Microsoft Word

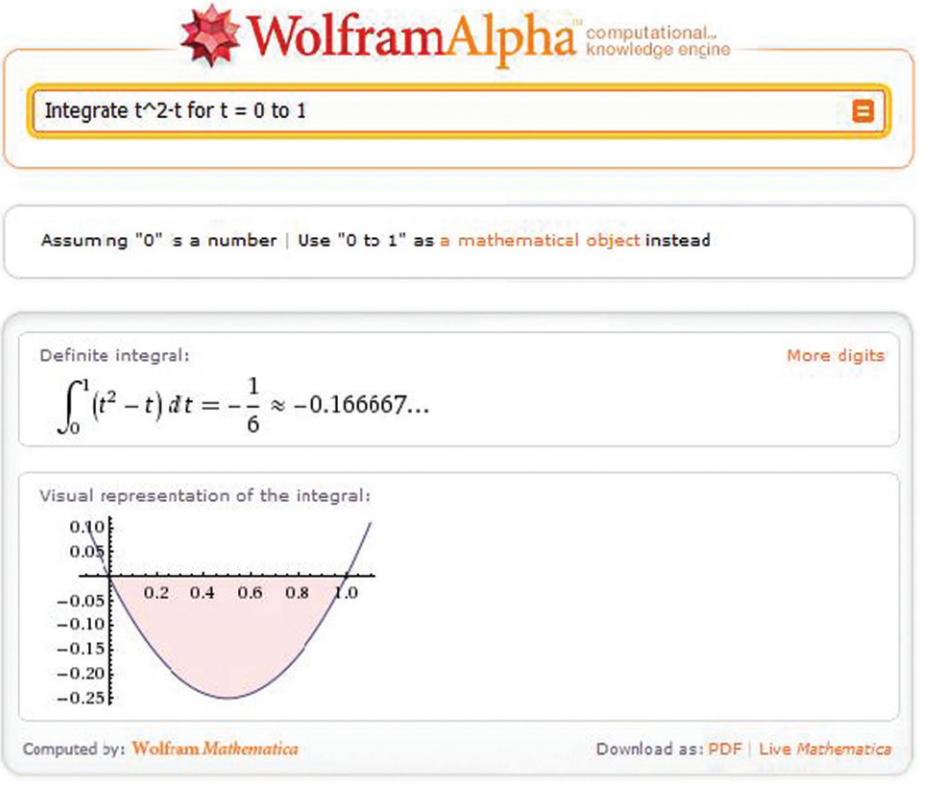

Figure 3. Integrate $t^2 - t$ for $t = 0$ to 1.

although they were given the option to use any word processing program. This worked well. The output from Wolfram|Alpha can be easily copied and pasted into the word processing document as an image. Also, the copyable plain text option for Wolfram|Alpha output allowed for multistep problems and investigating different examples of the same concept. The student did not have to type the input over again.

The Calculus Two class that participated in this project was comprised of 25 students. The students represent a variety of majors including mathematics, computer science, engineering, biology, chemistry and health science. Since one of the main goals of the project was to make the lab assignments more meaningful and accessible to the students, the Calculus Two students were asked to complete a brief survey on their experience. Nineteen students completed the survey. Many of the students in this course had a previous experience with lab assignments from Calculus One and 73% identified a specific CAS that they had used before this semester. Seventeen of the students responded that this was the first time that they had used Wolfram|Alpha and 15 students in the course selected that they felt comfortable using Wolfram|Alpha after the first lab assignment. That result is very encouraging since using our previous system, it would take several labs for the students to begin to feel comfortable.

From an educational standpoint, it was important to know if the students perceived that they had gained benefit in their learning of calculus as a result of the

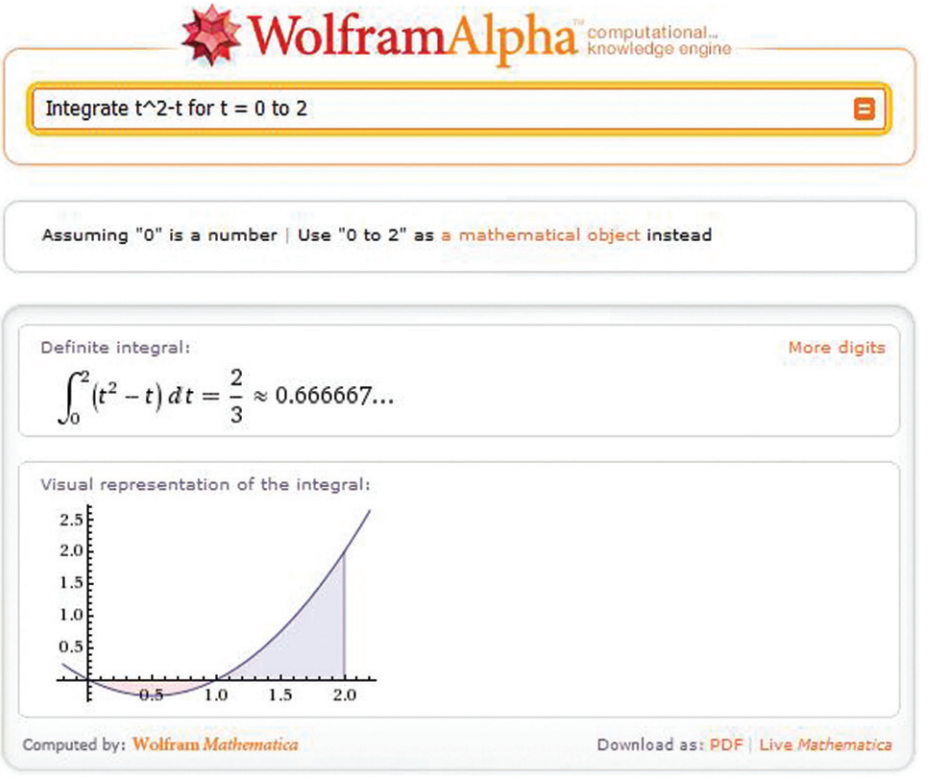

Figure 4. Integrate $t^2 - t$ for $t = 0$ to 2.

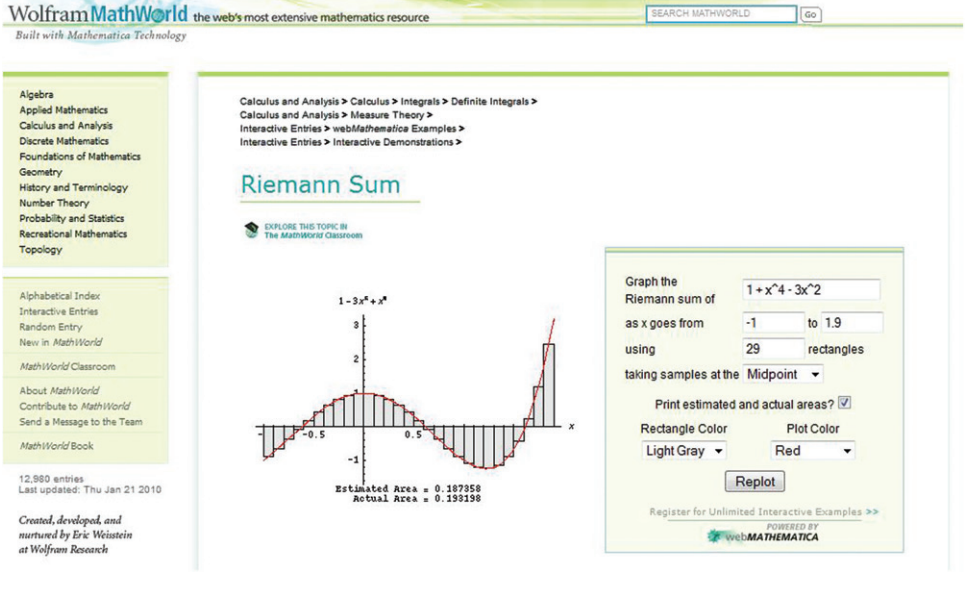

Figure 5. Riemann Sum activity from MathWorld.

lab assignments. Sixty-three per cent of the students indicated that Wolfram|Alpha was most helpful for assisting with computations, and 32% indicated that Wolfram|Alpha was most helpful for visualizing concepts. Another aspect to the project which was related to our collaboration with Wolfram Research was the idea that using Wolfram|Alpha for this purpose would increase the exposure and awareness of the site and possibly encourage future use of Wolfram|Alpha and other Wolfram products. To address this, the students were asked if they ever used Wolfram|Alpha outside of the lab assignments; in other words, did they ever choose to use it or did they only use it when it was required. Eleven students expressed that they had used Wolfram|Alpha several times outside of the lab assignments, and five expressed that they did but only once or twice. Overall, 63% of the students agreed that they would definitely use the site in future for other purposes, while 32% considered it a possibility.

The response from the students was very much in favour of continuing to use Wolfram|Alpha instead of going back to the previous CAS. The educational benefit of the lab assignments increased with the lack of emphasis on syntax. There are limitations as far as what can be done wholly in Wolfram|Alpha; however, MathWorld and the Wolfram Demonstration Project allow for helpful additions to fill in where Wolfram|Alpha is deficient.

5. Calculus of several variables (Calculus Three)

Wolfram|Alpha is becoming more useful in the multivariable context as time progresses. The main use of any CAS for multivariate calculus is for visualization. Three-dimensional graphing is somewhat limited in Wolfram|Alpha at this point in time; however, there has been some progress in 3-D graphing recently that will increase the usefulness of Wolfram|Alpha. In MathWorld, there are useful demonstrations of arc length, curvature and partial derivatives. Also, directional derivatives have a very helpful demonstration in MathWorld. However, it is still difficult to graph space curves and quadric (quadratic) surfaces. Also, parametric representations of surfaces are used extensively in vector calculus. These are very difficult to graph in Wolfram|Alpha. Some surfaces are not graphable at all. I was not able to utilize Wolfram|Alpha in the first few weeks of class, because the graphing I wanted was not easily accessible.

Unlike Calculus One and Two, in Calculus of Several Variables, the students cannot simply type in intuitively obvious commands to graph surfaces or space curves. These are the types of graphs that are necessary to illustrate the concepts in this third semester calculus course. Some common surfaces can be observed, such as quadratic surfaces. However, the students need to make connections between the different representations of these surfaces: algebraic (both in parametric form and in a form where the parameter has been eliminated), numerical and graphical. The first computer assignment in third semester calculus is one in which the students are to explore the effects of changing different parameters for quadratic surfaces. However, the student cannot input different forms of the equation for quadratic surfaces and see how the shape, position and orientation of the surfaces changes.

There are some excellent illustrations for some multivariable calculus concepts in the MathWorld Demonstration Project, for example, the Frenet Frame and Curvature of a space curve. However, again, these examples lack the freedom to change equations and parameters to demonstrate specific examples selected by the instructor or student. These types of explorations are invaluable in the learning experience of the third semester calculus student. Another illustration important to these students is the illustration of double Riemann sums for the approximation of the volume under a positive function, $z = f(x, y)$, over a (rectangular) region in the xy-plane.

So to conclude, I was unable to use Wolfram|Alpha for multivariate calculus labs, even if supplemented with Mathworld and Wolfram Demonstration Project, because it currently lacks the control and power of a full CAS that I require. It may indeed be possible at some point for me to migrate, as Wolfram|Alpha matures, but not at present.

6. Conclusion

We conclude that using Wolfram|Alpha as the computation engine for beginning calculus labs is not only possible but also offers advantages over standard CASs. These advantages include platform independence, natural language processing, being always up-to-date, the show details feature and cost (for both students and institutions). Our experience of converting our CAS-based labs over to Wolfram|Alpha was, in general, a good one. We do note, however, that Wolfram|Alpha is not as powerful as a standard CAS. These limitations make it currently difficult to replace a standard CAS with Wolfram|Alpha for classes beyond Calculus Two.

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